**Example 11-4: Transform of Impulse Train**

As another example of finding the Fourier transform of a periodic signal, let us consider the periodic impulse train

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad (11.41) \]

where the period is denoted by \( T_s \). This signal, which will be useful in Chapter 12 in deriving the sampling theorem, is plotted in Fig. 11-10(a). Because \( x(t) \) is periodic with period \( T_s \), we can also express (11.41) as a Fourier series

\[ p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad (11.42) \]

where \( \omega_0 = 2\pi / T_s \). To determine the Fourier coefficients \( \{a_k\} \), we must evaluate the Fourier series integral over one convenient period; i.e.,

\[
a_k = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t)e^{-j k \omega_0 t} \, dt
\]

\[
= \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) \, dt = \frac{1}{T_s}
\]

(11.43)

The Fourier coefficients for the periodic impulse train are all the same size. Now in general, the Fourier transform of a periodic signal represented by a Fourier series as in (11.42) is of the form

\[ P(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0) \]
Substituting (11.43) into the general expression for $P(j\omega)$, we obtain

$$P(j\omega) = \sum_{k=-\infty}^{\infty} \left( \frac{2\pi}{T_s} \right) \delta(\omega - k\omega_s)$$

(11.44)

Therefore, the Fourier transform of a periodic impulse train is also a periodic impulse train.