CS4115 Mid-Term Exam

Wed., Week07, 2012

- Please don't forget to fill in and sign your student ID on the grid; **do it now**
- Use the machine-readable multiple-choice question grid that has been provided to answer questions below; please completely mark in black exactly one circle on the grid for each answer
- To discourage guessing a penalty will be charged for incorrect answers; blank answers do not get penalised so if you are not very confident in an answer you might consider leaving the questions blank
- Exam is worth 15% of overall mark

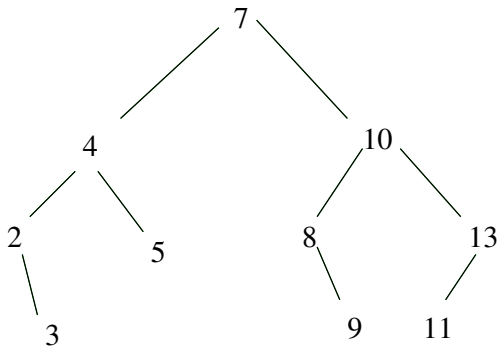

Figure 1: A Binary (Search) Tree

Section 1. Multiple Choice Questions (equal marks for each).

1. In a multiple choice exam, if negative marking is being used, what should be the penalty for wrong answers so that somebody who guesses throughout ends up with 0? Each question has four possible answers and the questions are worth 5 marks each.
   (a) -5
   (b) $-\frac{5}{4}$
   (c) $-\frac{5}{3}$
   (d) $-\frac{1}{3}$

2. Which of the answers below is $\sum_{i=0}^{\infty} 3^{-i}$?
   (a) $\frac{2}{3}$
   (b) ∞
   (c) $\frac{3}{2}$
   (d) None of the above

3. Which of the answers below is $\sum_{i=2}^{\infty} 3^{-i}$?
   (a) $\frac{2}{3} \times \frac{1}{3^2}$
   (b) $\frac{1}{6}$
   (c) ∞
   (d) None of the above

4. Which of the answers below is $\sum_{i=2}^{n+1} 6^i$?
   (a) $\frac{36}{5}(6^n - 1)$
   (b) $\frac{1}{5}(6^{n-1} - 1)$
   (c) $\frac{1}{5}(6^n - 1)$
   (d) None of the above

5. Which of the answers below is $\log_b x$?
   (a) $\log_2 b \times \log_2 x$
   (b) $\frac{\log_2 x}{\log_2 b}$
   (c) $\log_x 2$
   (d) $\log_x b$

6. Which of the answers below is $\sum_{i=1}^{50}(2i - 1)$?
   (a) 50^2
   (b) 51×50
   (c) 51×52
   (d) 49^2

7. Which of the answers below best approximates $\sum_{i=0}^{n} i^5$?
   (a) $O(i^5)$
   (b) $O(n^4)$
   (c) $O(i^6)$
   (d) None of the above

8. Which of the following best approximates $\sum_{i=1}^{\frac{n}{2}} \frac{1}{i}$?

   (a) $\frac{2}{n}$

   (b) $\frac{1}{n^2}$

   (c) $\log_e n$

   (d) $\log e$

9. Which of the following is a correct representation of $(a^b)^c$?

   (a) a^{b+c}

   (b) a^{bc}

   (c) a^{b^c}

   (d) a^{c^b}

10. Running-time $O(m+n)$ is equivalent to

    A $O(m) + O(n)$

    B $O(\max(m, n))$

    Which of these possiblities are correct?

    (a) Both **A** *and* **B**

    (b) Only **A**

    (c) Only **B**

    (d) Neither **A** *nor* **B**

11. What is the running-time in Big-Oh notation of the following chunk of code?

```
for (int i = 0; i < n; i++)
  i = n-i;
```

    (a) $O(n)$

    (b) $O(1)$

    (c) $O(n^2)$

    (d) $O(n \log n)$

12. What is the running-time in Big-Oh notation of the following chunk of code?

```
for (int i = 0; i < n*n; i++)
  sum += sum+i;
```

    (a) $O(n)$

    (b) $O(1)$

    (c) $O(n^2)$

    (d) $O(n \log n)$

13. Let $T_1(n) = O(f(n))$ and $T_2(n) = O(f(n))$. Given statements

    A $\frac{T_1(n)}{T_2(n)} = O(1)$;

    B $T_1(n) + T_2(n) = O(f(n))$

    which of them are true?

    (a) Both **A** *and* **B**

    (b) Only **A**

    (c) Only **B**

    (d) Neither **A** *nor* **B**

14. In a recent court case, a judge cited a city for contempt and ordered a fine of £2 for the first day. Each subsequent day, until the city followed the judge's order, the fine was squared (that is, the fine progressed £2, £4, £16, £256, £65,536, ...). What would be the fine on day n?

    (a) n^4

    (b) 2^{n^2}

    (c) $2^{2^{n-1}}$

    (d) 2^{2^n}

15. The time-complexity of exponentiation to the power of n is (most precisely)

    (a) $O(n)$

    (b) $O(\log n)$

    (c) $o(n)$

    (d) $\Theta(n)$

16. The time-complexity of searching for an item amongst an array of *sorted* items is (most precisely)

    (a) $\Theta(n \log n)$

    (b) $o(\log n)$

    (c) $O(\log n)$

    (d) $\Omega(\log n)$

17. What is the time-complexity of the following piece of code in "Big-Oh" notation?

```
sum = 0;
for (int i = 0; i < n; i++)
  sum += sum+n;
```

    (a) $O(n^2)$

    (b) $O(n)$

    (c) $O(\log n)$

    (d) $O(n \log n)$

18. What is the time-complexity of the following piece of code in "Big-Oh" notation?

```
sum = 0;
for (int i = 0; i < n; i++)
  for (j = 1; j < n; j = j*2)
    sum += n;
```

(a) $O(n^2)$
(b) $O(n)$
(c) $O(\log n)$
(d) $O(n \log n)$

19. What is the time-complexity of the following piece of code in "Big-Oh" notation?

```
sum = 0;
for (int i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    sum += n;
```

(a) $O(n^2)$
(b) $O(n)$
(c) $O(\log n)$
(d) $O(n \log n)$

20. What is the time-complexity of squaring a matrix of size $n \times n$ in "Big-Oh" notation?

(a) $O(n^3)$
(b) $O(n^2)$
(c) $O(n)$
(d) $O(n \log n)$

21. For the Binary Tree shown in Figure 1, which of the following represents a *post-order* traversal?

(a) **11,13,9,8,10,5,3,2,4,7**
(b) **7,4,10,2,5,8,13,3,9,11**
(c) **7,4,2,3,5,10,8,9,13,11**
(d) None of the above.

22. For the Binary Search Tree shown in Figure 1, deletion of node **7** would result in which of the following nodes becoming the root node?

(a) **4**
(b) **10**
(c) **4 or 10**
(d) **5 or 8**

23. For the Binary Search Tree shown in Figure 1, deletion of node **7** would result in the *inorder* traversal being which of the following?

(a) **4, 10, 2, 5, 8, 13, 3, 9, 11**
(b) **10, 4, 13, 8, 5, 2, 11, 9, 3**
(c) **–, 4, 10, 2, 5, 8, 13, 3, 9, 11**
(d) **2, 3, 4, 5, 8, 9, 10, 11, 13**

24. Which is larger, 18^9 or 9^{18}?

(a) 18^9
(b) 9^{18}
(c) They are both the same
(d) We didn't do a problem like this before

25. How many nodes are contained in a perfect ternary (3-way) tree of height k (levels $0 \ldots k$)?

(a) $\frac{3^k}{2}$
(b) $\frac{k^3-1}{2}$
(c) $\frac{3^{k+1}-1}{2}$
(d) 3^3

26. How many nodes are contained in the lowest layer of a perfect ternary (3-way) tree of height k (levels $0 \ldots k$)?

(a) $\frac{3^k}{2}$
(b) 3^k
(c) $\frac{k^3-1}{2}$
(d) $\frac{3^{k+1}-1}{2}$

27. How many nodes are contained in layers 10 to 23 of a perfect ternary (3-way) tree of height 30 (levels $0 \ldots 30$)?

(a) All three of the following answers
(b) $\frac{3^{24}-1}{2} - \frac{3^{10}-1}{2}$
(c) $\frac{3^{24}-3^{10}}{2}$
(d) $\frac{3^{10}(3^{14}-1)}{2}$

28. The Fibonacci numbers obey the recurrence relation

$$F_n = F_{n-1} + F_{n-2}, \quad n \geq 2, \quad F_0 = F_1 = 1$$

Which one of the following statements is **false**?

(a) $F_{n+1} = F_n + F_{n-1}, \quad n \geq 1$
(b) $F_{n+2} = F_{n+1} + F_n, \quad n \geq 0$
(c) $F_n = 1 + \sum_{i=0}^{n-2} F_i, \quad n \geq 2$
(d) $F_n = n, \quad n \geq 0$

In the following two questions `BST<Comparable>` is a Binary Search Tree `C++` class into which you may

insert "objects" that are comparable, that is, it is possible to decide amongst two which is larger and smaller. Each node of this tree will be a class, `BNode`, with a pointers to the left and right subtrees, `->left` and `->right`, respectively.

29. What does the function `what()` below, (or, more accurately, the member function `BST<Comparable>::what()`), perform?

```
template <class Comparable>
BNode<Comparable> *
BST<Comparable>::what( BNode<Comparable> *t )
{
  if( t == NULL )
    return NULL;
  if( t->left == NULL )
    return t;
  else
    return what( t->left );
}
```

    (a) Count the number of `NULL` nodes in the tree rooted at `t`

    (b) Find the leftmost node in the tree rooted at `t`

    (c) Find a node smaller than `t`

    (d) Find a node larger than `t`

30. What does the (member) function `whatEver()` below, perform? The function `max()` returns the larger of two numbers.

```
template <class Comparable>
int
BST<Comparable>::whatEver( BNode<Comparable> *t )
{
  if( t == NULL )
    return 0;

  int l = whatEver(t->left);
  int r = whatEver(t->right);
  int m = max(l, r); // larger of two ints
  return m+1;
}
```

    (a) Count the number of `NULL` nodes in the tree rooted at `t`

    (b) Count the number of non-`NULL` nodes in the tree rooted at `t`

    (c) Compute the height of the tree rooted at `t`

    (d) Compute the *age* of the tree rooted at `t`