University of Limerick

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Final Assessment Paper

Academic Year:	2015/2016	Semester:	Autumn
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	2 hours	Percent of Semester Marks:	60
Lecturer:	P. Healy	Paper marked out of:	60

Instructions to Candidates:

- There are two sections to the paper: Short Questions and Long Questions
- The mark distribution is 15 marks for Short Questions and 45 marks for the Long Questions
- Answer all questions in all sections

Section 1. Short Questions (5×3 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 1.1, 1.2, etc.
- Justify all answers; a justified answer is better than a guessed answer.

1. If two functions $f(n)$ and $g(n)$ are both $O(h(n))$ are the two functions asymptotically equal (differing by only a constant multiple)? That is, if $f(n) = g(n) = O(h((n))$ is it the case that $f(n) = O(g(n))$. True or false? Justify your answer.
2. How many passes of radix sort are required to sort an array of integers if we are limited to using 256 buckets on any pass? Does the size of integer play a role? Justify your answer.
3. What is the maximum no. of nodes possible in a d-ary heap of height k? For node number i, what is the number of its parent?
4. In a graph the degree of a vertex, d_v, is the number of neighbours v has (or the number of edges incident upon it). Call S the sum of all of the degrees, that is, $S = \sum_{v \in V} d_v$. Give an example of a graph $G = (V, E)$ where S is odd, or explain why there can not be such a G.

5. In the strict specification of the priority queue ADT the operation `decrease_p()`, which decreases the priority of a node in the queue but leaving it remain in the queue is not mentioned. Describe a situation where this operation could be of use.

Section 2. Long Questions (45 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 2.1, 2.2, and 2.3 in your answer books

1. Linked list management. **(15 marks.)**

   Polynomials can be efficiently represented using linked lists by storing as an item on the list each of the non-zero terms of the polynomial. So the polynomial $p(x) = 2x^3 - 7$ could be represented with a linked list of two items, where the first item is the pair $(2, 3)$ and the second is the pair $(-7, 0)$.

   Suppose we wish to multiply two polynomials $p(x)$ and $q(x)$ where $p(x)$ has m non-zero terms and $q(x)$ has n non-zero terms.

   (a) Give an example of two such polynomials whose product has mn terms. (3 marks.)

   (b) Give an $O(m^2n^2)$-time algorithm that multiplies two polynomials. Your answer, a linked list, should have one item on the list corresponding to one non-zero term of the multiplication, and no more. (6 marks.)

   (c) Give an $O(m^2n)$-time algorithm that multiplies two polynomials, where $q(x)$ is the shorter polynomial, that is, $m < n$. (4 marks.)

   (d) How likely is it that the product of two polynomials will have exactly mn unique terms? That is, if you are given a polynomial $q(x)$ can there always be a $p(x)$ that will guarantee that the product has the full mn terms? Explain how to construct such a $p(x)$ in the general case if yes. (2 marks.)

2. Palindrome Algorithms. **(15 marks.)**

   Wikipedia describes a Lychrel number as a positive integer "that cannot form a palindrome through the iterative process of repeatedly reversing its digits and adding the resulting numbers." Some examples of the process (also from Wikipedia) are

   - 56 becomes palindromic after one iteration: 56+65 = 121;
   - 59 becomes a palindrome after 3 iterations: 59+95 = 154, 154+451 = 605, 605+506 = 1111;
   - it takes 24 iterations for 89 to reach a palindrome;
   - after billions and billions of iterations it is still not known if the number 196 can ever become a palindrome, and 196 is the smallest integer unaccounted for; the status of 295 is also unknown.

Notice the asymmetry in how a Lychrel number is described. Once we find the palindrome we can definitely rule it out but for how long should we keep trying?

Suppose you are given the function `isPalindrome()` that checks whether the given argument is a palindrome and the function `reverseAndAdd()` that takes a number, reverses it, adds the two together and returns the result.

(a) Write a function `nonLychrel()` that, given a natural number, returns `true` if the argument is **not** a Lychrel number. What will happen if the argument given is 196? (5 marks.)

(b) Describe a function `belowLimit()` (pseudo-code is sufficient) that takes a single integer, `limit`, as argument and reports all of the non-Lychrel numbers smaller than the passed-in limit. For example, `belowLimit(300)` should print out all of the non-Lychrel numbers below 300.

Your function might never stop but it should report as many as possible when it is killed by the user. The order in which they are written out to the screen is not important. (5 marks.)

(c) Describe a function `findThemAll()` (pseudo-code is sufficient) that is planned to be let run forever that will print out all non-Lychrel numbers as they are discovered. (5 marks.)

3. Graph Algorithms. **(15 marks.)**

(a) Discuss how you could use an algorithm we have seen in labs to tell what vertices in a graph are exactly 4 edges away from a given vertex. (5 marks.)

(b) The Erdös Numbering Problem is to determine for a given computer scientist how *closely related* the researcher is to Paul Erdös, often considered to be one of the greatest computer scientists ever.

The Erdös Number (EN) of Erdös himself is 0; the Erdös number of an author is defined to be 1 if the author has co-written a paper in a scholarly journal with Erdös; otherwise, to determine an author's EN, find the lowest EN amongst all the people he has ever co-written a paper with and add 1 to that.

So, if author "Joe Bloggs" has EN 7, then "Seoirse de Blogg", a colleague and co-author of Joe Bloggs', will have EN *at most* 8. Note that his EN could possibly be lower than 8 if he has worked with somebody else who is more closely related to Erdös.

Using data structures and algorithms that we have studied in class develop an algorithm that reads a database of co-authors and determines the EN of some specified author. Your answer should be clear instructions along the lines of pseudo-code that could be turned into a program by a third person.

What is the worst-case running time of your algorithm? What are its space requirements? (10 marks.)