UNIVERSITY *of* LIMERICK

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FACULTY *of* SCIENCE *and* ENGINEERING

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2010/2011	Semester:	Spring
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	60
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

1. What is the time-complexity of the following piece of code in "Big-Oh" notation?

```
sum = 0;
for (int i = 0; i < n; i++)
  for (j = 1; j < n; j = j*2)
    sum = sum + n;
```

   (a) $O(n^2)$
   (b) $O(n \log n)$
   (c) $O(n)$
   (d) $O(\log n)$

2. The worst-case performances of the heap operations `deleteMin()` and `insert()` are both $O(\log n)$. Given the two statements below, which of them are true?

   S1 The experimentally found average case performance of `deleteMin()` is $O(1)$

   S2 The experimentally found average case performance of `insert()` is $O(1)$

   (a) **S1** is true, but **S2** is false
   (b) **S1** is false, but **S2** is true
   (c) Both statements are true
   (d) Both statements are false

3. The number of nodes in a *complete* binary tree of height h is
   - (a) exactly $2^{h-1} - 1$
   - (b) exactly $2^h - 1$
   - (c) exactly $2^{h+1} - 1$
   - (d) None of the above

4. How many nodes are on the bottom layer, h, of a *perfect* binary tree?
   - (a) exactly 2^h
   - (b) at least 2^h
   - (c) at most 2^h
   - (d) none of the above

5. Let $S_1 = \sum_{i=1}^{n} i^2$ and $S_2 = \left(\sum_{i=1}^{n} i\right)^3$. Which one of the following statements is true?
   - (a) $S_1 = S_2$ for $1 \leq n \leq 30$ only
   - (b) $S_1 = S_2$ for $1 \leq n \leq 100$ only
   - (c) $S_1 = S_2$ for all n
   - (d) None of the above

6. If $f(n) = O(g(n))$ which of the following statements cannot be true?
   - (a) $f(n) = o(g(n))$
   - (b) $g(n) = O(f(n))$
   - (c) $g(n) = \Theta(f(n))$
   - (d) $f(n) = \Theta(g(n))$

7. On the first day of Christmas,
   my true love sent to me
   A partridge in a pear tree.

   On the second day of Christmas,
   my true love sent to me
   Two Zetor tractors, and
   A partridge in a pear tree.

   On the third day of Christmas ...

   How many lines would be in such a "poem" if it ran for 365 days instead of the usual 12?
   - (a) $\frac{365 \times 366}{2} + 2 * 365$
   - (b) $\frac{367 \times 368}{2} - 3$
   - (c) Neither of the above
   - (d) Both of the above

8. Given the two statements below, which of them are true?

   S1 In a strongly connected graph, every node connects to every other node by a directed edge

   S2 If a graph is strongly connected then it cannot have a cut vertex (articulation point)
   - (a) **S1** is true, but **S2** is false
   - (b) **S1** is false, but **S2** is true
   - (c) Both statements are true
   - (d) Both statements are false

9. Given the two statements below, which of them are true?

   S1 If an n-vertex graph has n articulation points then the graph must have a cycle

   S2 If the Depth-First Tree of a graph G has no back edges then G has no cycles
   - (a) Both statements are true
   - (b) **S1** is true, but **S2** is false
   - (c) **S1** is false, but **S2** is true
   - (d) Both statements are false

10. Given the two statements below, which of them are true?

    S1 Starting from vertex v_0 in a graph, the time required by Depth-First Search to find a path (if one exists) to some vertex v^* is *less* than that required by Breadth-First Search

    S2 The space required by Depth-First Search is *less* than that required by Breadth-First Search
    - (a) **S1** is true, but **S2** is false
    - (b) **S1** is false, but **S2** is true
    - (c) Both statements are true
    - (d) Both statements are false

Section 2. Short Questions (5 × 4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.

1. The height of an AVL tree is no worse than _______ times the optimal height.
2. Give the recurrence relation for N_h, the number of nodes in the worst possible AVL tree of height h ______________________?
3. In a d-heap (a heap where each node can have at most d children), what are the locations of a node's children? The root node of the heap is at location 1. __________________
4. Sorting is possible in $o(n \log n)$-time with ____ sort.
5. If a graph has negative edge costs then the "Big-Oh" running time of the shortest path algorithm increases to ________.

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

1. Prime number detection. **(15 marks.)**

   (a) Write an efficient `C++` function to check if a given number is prime. What is the running time of your program? Your solution should differ from that given in the next part of the question since it asks something different. (5 marks.)

   (b) The code below does something different to the previous task. Called the *Sieve of Erastothenes*, it finds *all* prime numbers up to some given value. Explain *in detail* its working. (5 marks.)

   (c) Analyse its running time and give as good an estimate of its worst-case behaviour as you can. (5 marks.)

```
void genPrimes(int n)
{
  //create prime array
  bool *prime = new bool[n+1];

  //set 0 and 1 as not prime
  prime[0] = prime[1] = false;

  //set remainder 'true' initially
  for (int ind = 2; ind <= n; ind++)
    prime[ind] = true;

  //elimination time
  for (int p = 2; p <= (int)sqrt(n)+1; p++)
  {
    if(prime[p] == false) continue;

    for (int c = 2*p; c <= n+1; c = c+p)
      prime[c] = false; // multiple of p so composite
  }
}
```

2. Linked Lists. **(15 marks.)**
   Suppose you have two polynomials $P(x)$ and $Q(x)$ which are to be added together. Assume that $P(x)$ has n non-zero terms and that $Q(x)$ has m non-zero terms, where $n > m$. The polynomials are represented using linked lists for reasons of space efficiency and the lists should store the non-zero terms ordered from largest power to smallest power.

   (a) Draw a picture of the linked-list representation of $P(x) = -2x^{16} + 4x - 3$. (3 marks.)

   (b) Give an $O(n)$-time algorithm for performing the addition. Your algorithm should be written in `C++` or, failing that, pseudo-code. You may assume that your `Term` class has appropriate constructors. (9 marks.)

   (c) Is linear-time the best possible time asymptotically for part (b)? Why? (3 marks.)

3. Graph Algorithms. **(10 marks.)**
   The Erdös Numbering Problem is to determine for a given computer scientist how *closely related* the researcher is to Paul Erdös, often considered to be one of the great computer scientists.

   The Erdös Number (EN) of Erdös himself is 0; the Erdös number of an author is defined to be 1 if the author has co-written a paper in a mathematical journal with Erdös; otherwise, to determine an author's EN, find the lowest EN amongst all the people he has ever co-written a paper with and add 1 to that.

   So, if author "Joe Bloggs" has EN 7, then "Seoirse de Blogg", a colleague and co-author of Joe Bloggs', will have EN *at most* 8. Note that his EN could possibly be lower than 8 if he has worked with somebody else who is more closely related to Erdös.

   Using data structures and algorithms that we have studied in class develop an algorithm that reads a database of co-authors and determines the EN of some specified author.

   What is the worst-case running time of your algorithm? What are its space requirements?