



UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

FACULTY of SCIENCE and ENGINEERING

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2008/2009	Semester:	Spring
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	2½ hours	Percent of Semester Marks:	60
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

- | | |
|---|---|
| 1. In an AVL tree of height h , the number of probes made in an unsuccessful search will be | 2. Given $f(n) = n$ and $g(n) = 2n$, which one of the following statements is <i>false</i> . |
| (a) $O(1)$ | (a) $f(n) = O(g(n))$ |
| (b) $O(\log n)$ | (b) $f(n) = \Omega(f(n))$ |
| (c) $O(n)$ | (c) $f(n) = \Theta(f(n))$ |
| (d) Depends on the way the AVL was first created | (d) $f(n) = o(g(n))$ |

- How many multiplications are used in the calculation of X^{2^n} , using the “fast” exponentiation algorithm?
 - $O(\log n)$
 - $O(n)$
 - $O(2^n)$
 - $O(n!)$
- What is the vertex-connectivity of the graph drawn in Figure 1(a)?
 - 2
 - 3
 - 4
 - 5
- How many cut vertices does the graph drawn in Figure 1(b) have?
 - 1
 - 3
 - 4
 - 5
- On input of size N , the running time of programs A is always less than $212 \times N$. On input of size N , the running time of program B is always less than $212 \times \log(2^N)$. Is the average running time of program A greater than the average running time of program B , for $N = 100,000$.
 - Yes, the average running time of program A is greater than the average running time of program B
 - No, the average running time of program A equals the average running time of B , for $N = 100,000$
 - No, the average running time of program B is greater than the average running time of program A , for $N = 100,000$
 - There is not enough information to tell
- Consider what happens after 6 is inserted into the AVL tree drawn in Figure 2. The tree becomes unbalanced and needs to be re-balanced using a single-rotation. Following this single-rotation what node(s), if any, are the children of 6?
 - node 7 and node 8
 - just node 7
 - just node 8
 - node 6 will have no children
- Consider an arbitrary binary tree B where every node is either full or is a leaf. Let F represent the number of full nodes in B and let L represent the number of leaves in B . (Recall a full node is a node with two children, and a leaf is a node with no children). Which of the following statements is true, no matter what the choice of B ?
 - $F = L + 1$
 - $L = F + 1$
 - $L = 2F$
 - $F = 2L$
- For which of the following sorting algorithms, is it important that the input is random (rather than presorted or in reverse order for example)?
 - quicksort*, where the first element is chosen as the pivot
 - quicksort*, where the pivot is chosen using median-of-three partitioning
 - mergesort*
 - insertion sort*
- In an AVL tree of height h , the number of probes made in a *successful* search will be (most accurately)
 - $O(1)$
 - $O(\log n)$
 - $O(n)$
 - Depends on the way the AVL was created

Section 2. Short Questions (5×4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.
1. When solving the *weighted shortest path* a priority queue is used but uses a non-standard operation. What is this non-standard operation? _____.
 2. In O -notation, what is the analogue of a linear-time algorithm if the input is a graph $G = (V, E)$? _____.
 3. The height of an AVL tree is no worse than _____ times the optimal height.
 4. Given a graph, $G = (V, E)$, what is the largest number of edges exactly a graph can have in terms of $|V|$, the number of nodes? _____.
 5. Ordinarily the most appropriate way to represent a graph internally is with _____; however, if many queries are of the form “Is node u adjacent to node v ?” then the most appropriate representation may be _____.

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

1. Use induction to show that $n! > 2^n$, for $n > 3$. (10 marks.)
2. What does the function `what()` below return? Give a clear explanation referring to a worked example. Hint: recall the tutorial problem of decomposing an integer into its binary representation. (10 marks.)

```
int what(int a, int b)
{
    int p = 0;
    while (a != 0) {
        if (a%2 == 1) p += b;
        a = a / 2;
        b = b * 2;
    }
    return p;
}
```

3. (20 marks.)
 Given a graph $G = (V, E)$, where $n = |V|$ and $m = |E|$
 - (a) Show that every spanning tree of G has $n - 1$ edges. (8 marks.)
 - (b) Use the previous result to show that there are $m - (n - 1)$ different cycles in G . (6 marks.)
 - (c) A graph $G = (V, E)$ is called d -regular if every vertex has degree d . Figure 1 below illustrates a 4-regular graph. Does there exist a 3-regular graph G on 5 vertices? Either give an example or prove that one cannot exist. (6 marks.)

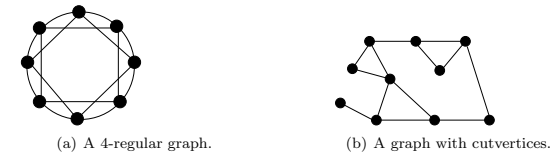


Figure 1: Some example graphs.

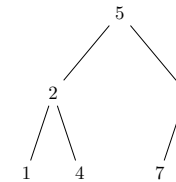


Figure 2: An AVL Tree.