



University of Limerick
Ollscoil Luimnigh

COLLEGE of INFORMATICS and ELECTRONICS

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2007/2008	Semester:	Spring
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	2½ hours	Percent of Semester Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

- | | |
|--|---|
| 1. The number of nodes in a <i>complete</i> binary tree of height h is | 2. How many nodes are on the bottom layer, h , of a <i>perfect</i> binary tree? |
| (a) exactly $2^{h-1} - 1$ | (a) at least 2^h |
| (b) exactly $2^h - 1$ | (b) at most 2^h |
| (c) exactly $2^{h+1} - 1$ | (c) exactly 2^h |
| (d) None of the above | (d) none of the above |

3. Let $S_1 = \sum_{i=1}^n i^2$ and $S_2 = (\sum_{i=1}^n i)^3$. Which one of the following statements is true?
- $S_1 = S_2$ for $1 \leq n \leq 30$ only
 - $S_1 = S_2$ for $1 \leq n \leq 100$ only
 - $S_1 = S_2$ for all n
 - None of the above
4. If $f(n) = O(g(n))$ which of the following statements cannot be true?
- $g(n) = O(f(n))$
 - $g(n) = \Theta(f(n))$
 - $f(n) = o(g(n))$
 - $f(n) = \Theta(g(n))$
5. On the first day of Christmas,
my true love sent to me
A partridge in a pear tree.

On the second day of Christmas,
my true love sent to me
Two Tsetse tsetse flies, and
A partridge in a pear tree.

On the third day of Christmas ...

How many lines would be in such a "poem" if it ran for 365 days instead of the usual 12?
- $\frac{365 \times 366}{2} + 2 * 365$
 - $\frac{367 \times 368}{2} - 3$
 - Neither of the above
 - Both of the above
6. What is the time-complexity of the following piece of code in "Big-Oh" notation?
- $O(n^2)$
 - $O(n)$
 - $O(\log n)$
 - $O(n \log n)$
7. The worst-case performances of the heap operations `deleteMin()` and `insert()` are both $O(\log n)$. Given the two statements below, which of them are true?
- S1 The experimentally found average case performance of `deleteMin()` is $O(1)$
- S2 The experimentally found average case performance of `insert()` is $O(1)$
- Both statements are true
 - S1 is true, but S2 is false
 - S1 is false, but S2 is true
 - Both statements are false
8. Given the two statements below, which of them are true?
- S1 In a strongly connected graph, every node connects to every other node by an edge
- S2 If a graph is strongly connected then it cannot have a cut vertex (articulation point)
- Both statements are true
 - S1 is true, but S2 is false
 - S1 is false, but S2 is true
 - Both statements are false
9. Given the two statements below, which of them are true?
- S1 If an n -vertex graph has n articulation points then the graph must have a cycle
- S2 If the Depth-First Tree of a graph G has no back edges then G has no cycles
- Both statements are true
 - S1 is true, but S2 is false
 - S1 is false, but S2 is true
 - Both statements are false
10. Given the two statements below, which of them are true?
- S1 Starting from vertex v_0 in a graph, the time required by Depth-First Search to find a path (if one exists) to some vertex v^* is *less* than that required by Breadth-First Search
- S2 The space required by Depth-First Search is *less* than that required by Breadth-First Search
- Both statements are true
 - S1 is true, but S2 is false
 - S1 is false, but S2 is true
 - Both statements are false

Section 2. Short Questions (5 × 4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.
1. The *unweighted shortest path* problem can be solved in _____ time.
 2. With $O(n)$ calls to `percolate_down()`, a heap can be created in _____ time.
 3. Give the recurrence relation for the *best-case* running time of `QuickSelect()`, the algorithm for finding the k^{th} largest element in an array: _____
 4. Recursion is to algorithm implementation as _____ is to proof techniques. That is, what is the proof technique analogue of recursion? _____
 5. Ordinarily the most appropriate way to represent a graph internally is with _____; however, if many queries are of the form “Is node u adjacent to node v ?” then the most appropriate representation may be _____

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

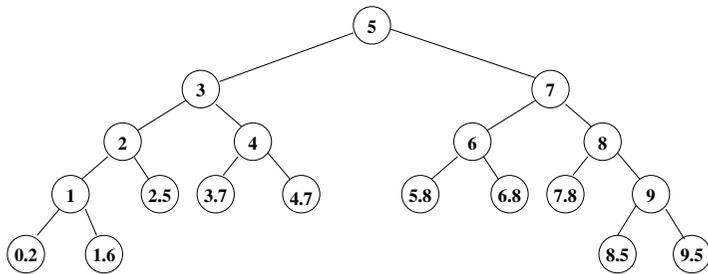


Figure 1: A Binary Search / AVL Tree

1. Consider the tree shown in Figure 1. (10 marks.)
 - (a) Assuming that the tree is a Binary Search Tree, delete the node numbered 5; (5 marks.)
 - (b) Assuming that the tree is an AVL tree, insert the number 8.8. (5 marks.)
2. (15 marks.)
 - (a) In arguing that any sorting algorithm that relies on 2-way comparisons requires $\Omega(n \log n)$ comparisons we claimed that a binary tree with L leaves had to have depth at least $\lceil \log L \rceil$. Use induction to show that the number of leaves, $|L_d|$, of a binary tree of depth d is at most $|L_d| \leq 2^d$. (7 marks.)

- (b) Draw the height-4 AVL tree that has the minimum number of nodes. That is, draw the worst, most skewed tree that has depth, $d = 4$. (3 marks.)
- (c) Give the recurrence relation for n_d the smallest number of nodes possible in an AVL tree of depth d . What are the initial conditions? (5 marks.)

3. (15 marks.)

The vertex connectivity problem that we have looked at in class has a wide variety of applications. The *vertex connectivity* problem is to determine the number of edge-disjoint paths that exist between pairs of nodes in G .

- (a) What is the vertex-connectivity of the graph shown in Figure 2 over? (4 marks.)
- (b) Give an *upper* bound, k_u , on the connectivity of a graph, G in terms of its nodes and edges; (4 marks.)
- (c) Prove that no graph can have higher connectivity than your bound k_u above. (7 marks.)

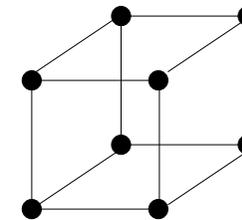


Figure 2: What is the vertex-connectivity of this graph?