UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE of INFORMATICS and ELECTRONICS

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2006/2007	Semester:	Autumn
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are two sections to the paper: Short Questions and Long Questions
- The mark distribution is 20 marks for Short Questions and 80 marks for the Long Questions
- Answer all questions in all sections
- Also, **please ensure** that marks posted on the class web page for all in-semester assessments coincide with your expectation

Section 1. Short Questions (5 × 4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 1.1, 1.2, etc.

1. *Dijkstra's Algorithm* requires __________ edge costs.
2. *Dijkstra's Algorithm* runs in time __________.
3. If a divide-and-conquer algorithm is of the form
$$T(1) = b$$
$$T(n) = aT(\frac{n}{c}) + bn$$
then the worst-case running time of the algorithm when $a < c$ is, in Big-Oh notation, ____.
4. Computing
$$S = \sum_{i=0}^{n} i^i$$
can be done in running time, in Big-Oh notation, _____. Note this question is not about the *value* of S; rather, it is about the time taken to compute S.
5. For *open hashing* it is advisable that λ, the load factor, be no larger than ___.

Section 2. Long Questions (80 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 2.1, 2.2, 2.3, and 2.4 in your answer books

1. **(20 marks.)**

   (a) Evaluate the sum (10 marks.)

$$\sum_{i=0}^{\infty} \frac{i}{4^i}$$

   (b) Prove the following formula (10 marks.)

$$\sum_{i=1}^{n} i^3 = \Big(\sum_{i=0}^{n} i\Big)^2$$

2. **(20 marks.)**

   (a) Give the recurrence relation for the `mergesort` algorithm. (4 marks.)

   (b) Solve the general recurrence relation (10 marks.)

$$T(1) = b$$
$$T(n) = aT(\frac{n}{c}) + bn$$

   (c) As a special case of the above, or by any other means, solve the recurrence relation for the `mergesort` algorithm. (6 marks.)

3. **(20 marks.)**

   (a) Show the result of inserting $3, 1, 4, 6, 9, 2, 5, 7$ into an initially empty binary search tree. (8 marks.)

   (b) Show the result of deleting the root. (4 marks.)

   (c) Now repeat the insertions of the first part using an AVL tree this time. (8 marks.)

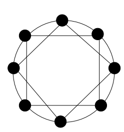
4. **(20 marks.)**

   Given a graph $G = (V, E)$, where $n = |V|$ and $m = |E|$

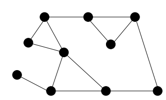
   (a) Show that every spanning tree of G has $n - 1$ edges. (8 marks.)

   (b) Use the previous result to show that there are $m - n + 1$ different cycles in G. (6 marks.)

   (c) A graph $G = (V, E)$ is called d-regular if every vertex has degree d. Figure 1 below illustrates a 4-regular graph. Does there exist a 3-regular graph G on 5 vertices? Either give an example or prove that one cannot exist. (6 marks.)



(a) A 4-regular graph.



(b) A graph with cutvertices.

Figure 1: Some example graphs.