UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE *of* INFORMATICS *and* ELECTRONICS

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2005/2006	Semester:	Autumn
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are two sections to the paper: Short Questions and Long Questions
- The mark distribution is 20 marks for Short Questions and 80 marks for the Long Questions
- Answer all questions in all sections
- **You *must* return this paper with your answer book**

Section 1. Short Questions (5 × 4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 1.1, 1.2, etc.

1. The *unweighted shortest path* problem can be solved in _____ time.
2. With $O(n)$ calls to `percolate_down()`, a heap can be created in ____ time.
3. Give the recurrence relation for the *best-case* running time of `QuickSelect()`, the algorithm for finding the k^{th} largest element in an array: _____
4. Recursion is to algorithm implementation as _____ is to proof techniques. That is, what is the proof technique analogue of recursion?
5. Ordinarily the most appropriate way to represent a graph internally is with _____; however, if many queries are of the form "Is node u adjacant to node v?" then the most appropriate representation may be _____

Section 2. Long Questions (80 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 2.1, 2.2, 2.3, and 2.4 in your answer books

1. **(20 marks.)**

   (a) When proving a lower bound on sorting algorithms we had cause to consider $\log(n!)$. Use Stirling's formula,

   $$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

   to give a precise estimate for $\log(n!)$. (5 marks.)

(b) Suppose you have an array of n elements containing only two distinct keys, `true` and `false`. Give an $O(n)$ algorithm to rearrange the array so that all `false` elements precede all `true` elements.
You may use only constant extra space. (6 marks.)

(c) Suppose you have an array of n elements containing three distinct keys, `true`, `false` and `maybe`. Give an $O(n)$ algorithm to rearrange the array so that all `false` elements precede all `maybe` elements and all `maybe` elements precede all `true` elements.
You may use only constant extra space. (9 marks.)

2. **(20 marks.)**

(a) Show the result of inserting $10, 12, 1, 14, 6, 5, 8, 15, 3, 9, 7, 4, 11, 13\text{and}2$, one at a time, into an initially empty binary heap. (7 marks.)

(b) Show the result of using the linear-time algorithm `build_heap()` to build a binary heap using the same input. (7 marks.)

(c) Show the result of performing three `deleteMin()` operations in the heap of the previous example. (6 marks.)

3. **(20 marks.)**

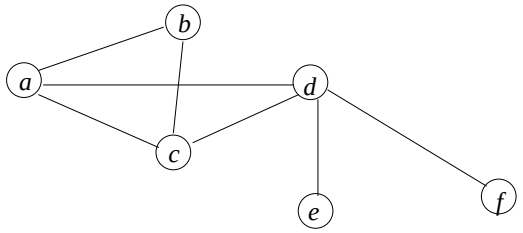
(a) What is an *articulation point* or *cut vertex* in a graph? (3 marks.)

(b) What is a graph called that has *no* cut vertices? (3 marks.)

(c) Given the graph $G = (V, E)$ shown in Figure 1 below, show its DFS tree, starting the search from a and proceeding lexicographically whenever a choice of edge exists. (6 marks.)

(d) For the DFS tree you have drawn, compute its *num* and *low* numbers. (6 marks.)

(e) When testing a graph for cut vertices what is the property in terms of *num* and *low* numbers that guarantees existence of a cut vertex? (2 marks.)



Graph, G

Figure 1: A graph G.

4. **(20 marks.)**

An undirected graph, $G = (V, E)$, is a called a *cubic* graph if *every* vertex has degree 3 and is connected (one connected component).

(a) What is the smallest cubic graph? (3 marks.)

(b) Argue that there cannot be an n-vertex cubic graph when n is odd. (3 marks.)

(c) What is the largest cubic graph and why? (4 marks.)

(d) A Hamiltonian[1] cycle is a cycle of length n that visits every vertex exactly once. Prove that every cubic graph has a Hamiltonian cycle. (10 marks.)

[1]Name after William Hamilton, Ireland's most famous mathematician.